Application of Design-of-Experiment Methods and Surrogate Models in Electromagnetic Nondestructive Evaluation

**Thesis points of the PhD Dissertation**
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1 Introduction

1.1 Forward and inverse problems in electromagnetic nondestructive evaluation

Electromagnetic Nondestructive Evaluation (ENDE) is applied in various industrial domains for the exploration of hidden in-material defects of structural components. The methods rely on the fact that the electromagnetic (EM) constitutive parameters of the material are locally changed in the presence of a defect. Based on the measured EM field (generated by an external source and interacting with the examined specimen), ENDE aims at the characterization of the defects. Among others, for instance, Eddy-Current Testing (ECT) is a popular ENDE method for the examination of conductive specimens. During the inspection, eddy-currents are generated within the specimen, and the magnetic field is measured outside. The latter carries information on the distribution of the eddy-currents, so on the material structure as well. In Fig. 1 a simple setup is illustrated, where a single coil is driven by alternating current, and the variation of its impedance –due to the defect– is measured at different coil positions.

![Figure 1: Cross-section sketch of a simple ECT setup and the EM field.](image)

In mathematical terms, the defect characterization task of ENDE is an inverse problem, i.e., it consists in the determination of the physical configuration based on the observations of the corresponding EM field. To perform inversion, one has to be able to determine the EM field corresponding to a known defect, i.e., to solve the forward problem. Practically, this is achieved via the mathematical modeling (based on the Maxwell’s equations) and the numerical simulation (e.g., by the Method of Moments) of the studied ENDE setup. Such simulators can provide fine precision, but at a price of com-
putational cost. However, the solution of an inverse problem often requires several runs of these “expensive-to-evaluate” simulators, making the inversion procedure firmly demanding in terms of runtime and computational resources.

Concerning the inverse problem, one usually has to face with its ill-posedness: the solution of the inverse problem (in contrary with the forward case) might not exist, might not be unique and might be unstable. For these reasons, one often needs a sort of characterization of this ill-posedness as well, e.g., the description of the effect of the measurement noise on the uncertainty of the defect reconstruction.

1.2 Surrogate modeling and design-of-experiments

To overcome the challenges arisen by the computational cost of the numerical EM simulators, surrogate modeling (SM) is getting more and more widespread in electromagnetics. A surrogate model imitates the true model, but as a rule, it is much less complex than the latter. A way to construct such surrogates is to perform a couple of simulations (i.e., to “sample” the given problem) and then to approximate the model based on the obtained data. The interpolation-based schemes are widely used: by fitting an interpolator to the observed data, a simple surrogate model can be achieved, which does not account for the inner structure of the EM simulator at hand, i.e., the latter is treated as a black-box.

The precision of the yielded interpolation-based surrogate model (e.g., in terms of mean-squared discrepancy between the true and interpolated data) strongly depends on both the number of samples and the chosen interpolator. As a rule, one must consider a sophisticated strategy –drawn from the tools of Design-of-Experiments (DoE)– for the appropriate choice of the “prototype” simulations, in order to obtain the “best” surrogate model within a given budget of EM simulations. Recently, adaptive DoE methods have been developed and are still in the focus of research. Such approaches take into account both the modeled problem and the goal of the subsequent use of the simulated data. As a rule, the adaptive DoE methods are sequential sampling schemes, and the choice of the next observation depends on the previously obtained data.

In the surrogate models, nowadays, kriging –initially developed in geostatistics, in the 1960s– is a popular generic interpolator. Based on some observations of an unknown scalar function, the latter can be interpolated by using kriging. The main idea is to model the unknown function by a Gaussian random process, which is then predicted as a sort of “optimal” linear combination of the observations. The covariance function of the modeling process are
usually to be estimated from the observed function values. Beyond the mere interpolation, kriging provides the estimated uncertainty of the approximation as well, in terms of the variance of the prediction error.

1.3 General goal and frame of the research

The objective of the research presented in this Dissertation is to improve the performance of ENDE methods by involving DoE and SM approaches. To this end, we apply existing DoE/SM methods and also devise new ones. Although the methods are presented in the framework of ENDE, we believe that a more general use is also possible in most of the cases, since the forward problem is treated as a black-box.

The research have not been crowned by industrial applications yet, however, we see some potential possibilities (pointed out in the next section) for this. The inspiration of our work –namely, the need for fast and reliable surrogate models to solve the forward and inverse problems in ENDE– is indeed a real claim of the industry.

We illustrate the presented approaches by a specific ECT setup. We assume parametric models (using 2, 4, 6 and occasionally 8 parameters) of one single or a pair of parallel thin, rectangular-shaped cracks within a conductive plate. These parameters are referred as the input of the model in the following. The set of all feasible input is the input space. The measured data –called the output of the model– are the variation of the coil impedance in function of the coil position over a rectangular region. The input-output relationship is formalized via the forward operator, being realized by an EM simulator of the studied ECT setup. The EM modeling and simulation issues are only slightly concerned in the Dissertation: a surface integral equation-based defect model is numerically simulated by using a moment’s method discretization scheme.

2 Summary of the new scientific achievements

In this section, the three main research topics –forming three different chapters in the Dissertation– are briefly presented. At the beginning of each item, the main contribution is formulated in a consistent sentence as a “thesis point”, then a short explanation is given.
2.1 Inversion by the “Efficient Global Optimization” algorithm

I have devised an inversion method for nondestructive evaluation by combining a numerical simulator of the electromagnetic phenomena and the “Efficient Global Optimization” (EGO) algorithm, which inversion scheme needs only a small number of forward simulations to solve an inverse problem even if the number of parameters to be retrieved is relatively high.

A classical inversion method is to compare the measured data to the output of an appropriate simulator and to reduce the discrepancy between the measured and simulated data. The solution of the (regularized) inverse problem is then those combinations of the input parameters of the simulator which yield the “smallest” discrepancy (in terms of an appropriate objective function, e.g., the mean-squared data misfit). One of the main bottlenecks of this optimization-based inversion approach is the computational cost of the numerical simulator. Thus, on one hand, the number of simulations needed to solve an inverse problem should be small. On the other hand, however, the yielded objective function to be minimized might be multimodal (possibly referring to the ill-posedness of the inverse problem), claiming the use of a global optimization method.

I have applied the “Efficient Global Optimization” (EGO) algorithm for the first time in a classical optimization-based inversion scheme to solve eddy-current testing inverse problems. The EGO algorithm has been designed for the optimization of expensive-to-evaluate objective functions as the number of observations needed to obtain the global minimum is usually small (compared to certain stochastic global optimization strategies). The algorithm consists in a sequential sampling strategy (being indeed an adaptive DoE tool), based on the kriging interpolation (i.e., the surrogate model) of the objective function. After the initial observations, new evaluations of the objective function are performed one-by-one. The choice of the next input depends on a criterion balancing between the local and global minimum search: the “promising” regions (i.e., where small objective function values are predicted by the kriging model) and the “unexplored” domains of the input space (i.e., where the uncertainty of the kriging prediction is high) are both raked over.

To sum up the advantages of the approach, we can emphasize (i) the relatively small number of forward simulations needed to solve the inverse problem and (ii) the convergence of the EGO algorithm to the global minimum of the objective function. Further advantages are provided by the implementation based on a coordinatewise search algorithm on a discrete grid (covering the
input space): (iii) the required precision of the solution of the inverse problem can be taken into account to some extent; (iv) the spatial discretization used in the integral equation-based numerical simulator can also be taken into account in the inversion scheme, thus, considerable computation time can be saved.

This research is reported in the following publications:

- [2] (peer-reviewed journal paper): the characterization of a volumetric defect;
- [7] (conference paper): characterization of a pair of thin cracks;
- [8] (conference paper): a brief summary with illustration of thin crack inversion;

2.2 Surrogate modeling by functional kriging

I have devised a surrogate modeling method for nondestructive evaluation which uses functional kriging to interpolate the forward operator based on a set of observations chosen adaptively in order to improve the precision of the surrogate model being built.

As mentioned in the Introduction, a natural way of constructing a fast surrogate model of a given forward operator is to perform evaluations at certain well-chosen input values, then to fit an interpolator to the observed data. The main benefit of such methods is the separation of the time-consuming task of the “database generation” and the subsequent use of the surrogate model by means of a fast interpolation. The first task is performed by “experts”, equipped with strong computational resources, the complex EM simulation code, etc. In the contrary, the “end-user” (which might neither have time to run overlong simulations, nor be expected to know the EM modeling and simulation in detail), only has to recourse to the pre-calculated data. In so doing, a problem-specific “off-line” simulator can be delivered to the end-user.

The central questions of such interpolation-based surrogate modeling are (i) the choice of the interpolator and (ii) the choice of the samples on which the interpolation is based, in order to obtain a “good” surrogate model, i.e., small discrepancy between the true and interpolated output data all over the input space.
I have applied the functional kriging technique to interpolate of the output data of an EM simulator. Functional kriging –a recent extension of the original kriging theory– makes possible the prediction of whole functions as single entities, instead of pointwise predictions. Since the models involved in electromagnetic nondestructive evaluation often have a functional output, the proposed approach is peculiarly convenient.

I have pointed out that in certain cases, the interpolation error of functional kriging can be considerably reduced by choosing the input samples adaptively, rather than using a regular grid (a so-called factorial design) in the input space. Apropos of this, I have proposed an adaptive sampling strategy, aiming at the step-by-step improvement of the surrogate model being built. The algorithm is driven by the estimated variance of the error of the kriging prediction, which can be considered as a sort of estimation for the interpolation error. To estimate the variance, I have used jackknifing –being a standard statistical technique–, and the sampling task is formalized as an optimization problem.

This surrogate modeling approach must have the potential for industrial use. Indeed, our industrial partner, the Commissariat à l’Énergie Atomique (CEA, France) manifests certain interest in implementing such strategies in a commercial software for nondestructive evaluation. One might also imagine to store the database in the memory of a small embedded measurement system, which is able to perform a sort of inversion based on the interpolated data. In so doing, a compact, problem-specific instrument for ENDE inversion might be yielded.

This research is reported in the following publications:

- [4] (peer-reviewed journal paper): the use of functional kriging to predict the output data of thin crack ECT models based on a full-factorial design;
- [9] (conference paper): the extension of the previous contribution with the adaptive sampling strategy;
- [12] (accepted conference paper): an application of the functional kriging-based surrogate model to build a new surrogate for “inverse interpolation”;
- [13] (accepted conference paper): a brief summary in the frame of an overview of kriging-based surrogate modeling;
- [14] (accepted conference paper): functional kriging-based surrogate modeling applied to the direct problem of radar observations of forested areas.
2.3 Output space-filling databases and inverse mappings

I have devised an adaptive sampling method for the generation of output space-filling defect-databases for nondestructive evaluation, along with two techniques to exploit certain meta-information provided by such output space-filling databases for the quantitative characterization of the related inverse problem.

A database consisting of input parameter - output data pairs can be considered as a sort of discrete representation of the involved forward operator. Such databases can form the basis of cheap surrogate models, by fitting an interpolator to the stored samples (as discussed in the explanation of the previous thesis point).

Classical Design-Of-Experiment provides numerous tools to generate sample sets satisfying certain criteria related to the repartition of the input samples (e.g., space-filling, uniformity). My main contribution in the frame of this thesis point is the extension of the input space-filling to the output space (i.e., to the domain of the output data – impedance signals in ECT – corresponding to each feasible input) in a certain sense. I have devised a sampling strategy which aims at building a database whose output samples fill the output space evenly: (i) none of the output samples is “too close” to an other one; (ii) for all elements of the output space, an output sample exists “not too far”.

The benefit of such output space-filling (OSF) databases is twofold: (i) generally, the interpolation error of the nearest neighbour interpolator is smaller compared to using input space-filling databases; (ii) the repartition of the input samples carries some meta-information on the modeled forward operator (e.g., indicates the regions where the forward operator is flat or varies rapidly).

The sampling strategy consists in the alternating use of subsequent sample insertion and removal, driven by the pairwise distances of the output samples. To control these distances, I have introduced the distance functions, being defined over the input space but expressing distances in the output space. The sample insertion criterion can thus be formalized as an optimization problem. To reduce the computational cost of the procedure, the kriging interpolation is used to approximate the distance functions.

Based on the distance functions and their kriging interpolation, I have devised two methods for the quantitative characterization of the underlying inverse problem. The first method aims at the inverse mapping of a given noise level from the output space to the input space. The inverse-mapped noise level characterizes the ill-posedness of the inverse problem by indicating how the uncertainty of the measured output influences the uncertainty of the
reconstructed input parameters. The second method maps the Voronoi cells in the output space into the input space. To a certain extent, the inverse-mapped Voronoi cells can be considered as the representation of the “resolution” provided by the studied database when addressing the inverse problem.

An interesting study has also been carried out: the database generation strategy has been applied to the forward problem of radar observations of forested areas. The input parameters were the frequency and the incident angle of the incident wave, the output data consisted of certain quantities derived from the scattered EM field (back-scattering coefficient, attenuation, interferometric height).

This research is reported in the following publications:

- [1] (peer-reviewed journal paper): basically contains all that we present in the corresponding chapter of the dissertation;
- [3] (peer-reviewed journal paper): a former and the present version of the OSF database generation, without the inverse mappings;
- [5] (peer-reviewed paper in edited book): a former version of database generation (without the sample-removal step);
- [6] (conference paper): focuses on the inverse mappings and the characterization of the inverse problems;
- [10] (conference paper): a shorter version of the previous contribution;
- [11] (conference paper): application of the OSF sampling to forest characterization;
3 Publications related to the thesis points

Papers in international peer-reviewed journal


Paper in edited book


Papers in proceedings of international conference


Papers accepted for international conference

